WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2022-23

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question Number 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Examine whether the limit $\lim\limits_{x\to3}\frac{[x]}{x}$ exists, where $[x]$ represents the greatest integer less or equal to x.

(b) $f(x)=\begin{cases} x+1 & \text{when } x\le1 \\ 3-ax & \text{when } x>1 \end{cases}$

For what value of a, will f be continuous at $x=1$.

(c) For the function $f(x)=|x|;\ x\in\mathbb{R}$ show that $f'(0)$ does not exists.

(d) Show that the function $f(x)=4x^2-6x-11$ is increasing at $x=4$.

(e) Find the point on the curve $y=x^3-6x+7$ where the tangent is parallel to the straight line $y=6x+1$.

(f) Find the asymptotes of the curve $xy^2-yx^2-(x+y+1)=0$.

(g) Examine the continuity of the function at $(0, 0)$

$$f(x,y)=\begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

(h) Show that the function $f(x,y)=\dfrac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$ is homogeneous in x and y. Find its degree.

(i) If $u=x\log y$, then show that $u_{xy}=u_{yx}$.

2. (a) If f is an even function and $f'(0)$ exists, then show that $f'(0)=0$. 4

(b) Discuss the continuity of f at $x=1$ and $x=2$ where $f(x)=|x-1|+|x-2|$. 4

3. (a) If $x+y=e^{x-y}$, show that $\dfrac{d^2y}{dx^2}=\dfrac{4(x+y)}{(x+y+1)^3}$ 4

(b) State and prove Lagrange's Mean Value Theorem. 4

4. (a) Find the slope of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ at the point (x_1, y_1) and hence obtain the equation of the tangent at that point. 4

(b) Verify Rolle's theorem for the function $f(x)=x\sqrt{4-x^2}$ is $0\le x\le 2$. 4

5. (a) Expand $f(x)=\sin x$ as a series of infinite terms. 5

(b) If $y=\dfrac{x}{x+1}$, show that $y_5(0)=51$. 3

6. (a) If $f(x)=\log\dfrac{\sqrt{a+bx}-\sqrt{a-bx}}{\sqrt{a+bx}+\sqrt{a-bx}}$, find for what values of x, $\dfrac{1}{f'(x)}=0$. 4

(b) Prove that $\lim\limits_{h\to 0}\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2}=f''(a)$, provided that $f''(x)$ is continuous. 4

7. (a) Find the maxima and minima, if any, of $\dfrac{x^4}{(x-1)(x-3)^3}$. 2+2

(b) Determine the values of a, b, c so that $\dfrac{a\sin x-bx+cx^2+x^3}{2x^2\log(1+x)-2x^3+x^4}$ may tend to a finite limit as $x\to 0$, and determine this limit. 3+1

8. (a) If $lx+my=1$ is a normal to the parabola $y^2=4ax$, then show that $al^3+2alm^2=m^2$. 4

(b) If the tangent at (x_1, y_1) to the curve $x^3+y^3=a^3$ meets the curve again in (x_2, y_2), show that $\dfrac{x_2}{x_1}+\dfrac{y_2}{y_1}=-1$. 4

9. (a) Prove that the asymptotes of the curve $x^2y^2=a^2(x^2+y^2)$ form a square of side $2a$. 4

(b) Show that for an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, the radius of curvature at an extremity of the major axis is equal to the half of the latus rectum. 4

10.(a) If V is a function r alone, where $r^2=x^2+y^2+z^2$, show that 4

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=\frac{d^2V}{dr^2}+\frac{2}{r}\frac{dV}{dr}.$$

(b) If $y=f(x+ct)+\phi(x-ct)$, show that $\dfrac{\partial^2 y}{\partial t^2}=c^2\dfrac{\partial^2 y}{\partial x^2}$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2021-22

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Does $\lim\limits_{(x,\,y)\to(0,\,0)} \dfrac{2xy^3}{x^2+y^6}$ exist? Give reasons. 2

(b) Use ε-δ definition of the limit to prove $\lim\limits_{x\to -3} x^2 = 9$. 2

(c) Find the coordinates of the points on the curve $y = x^3 - 6x + 7$ where the tangent is parallel to $y = 6x + 1$. 2

(d) Find domain of the function $f(x) = \sqrt{x-1} + \sqrt{5-x}$. 2

(e) Is Rolle's theorem applicable for the function $f(x) = x^2 - 5x + 6$ in [1, 4]? Justify your answer. 2

(f) Evaluate $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$. 2

(g) Prove that the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \sin x$, $x \in \mathbb{R}$ is continuous on $\mathbb{R}$ by using the ε-δ definition of continuity. 2

(h) Examine the nature of discontinuity of the function f defined by 2

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{x}} & x > 0 \\ 0 & x = 0 \end{cases}$$

at 0.

(i) Find the curvature of the parabola $x^2 = 12y$ at the point $\left(-3, \frac{3}{4}\right)$. 2

2. (a) A function f in [0, 1] is defined as follows 5

$$\begin{aligned} f(x) &= x^2 + x \quad &, \quad 0 \le x < 1 \\ &= 2 \quad &, \quad x = 1 \\ &= 2x^3 - x + 1 \quad &, \quad 1 < x \le 2 \end{aligned}$$

Examine the differentiability of f at $x = 1$. Is f continuous at $x = 1$?

(b) If $f: I \longrightarrow \mathbb{R}$ is a function differentiable at a point $c \in I$, then show that it is continuous at c. 3

3. (a) If $x = \sec\theta - \cos\theta,\ y = \sec^n\theta - \cos^n\theta$, show that $(x^2+4)\left(\dfrac{dy}{dx}\right)^2 = n^2(y^2+4)$. 4

(b) If $\lim\limits_{x\to 0} \dfrac{a\sin x - \sin 2x}{\tan^3 x}$ is finite, find the value of a and the limit. 4

4. (a) If $f(x) = \sin x$, find the limiting value of θ, when $h \to 0$ using the Lagrange's mean value theorem $f(a+h) = f(a) + hf'(a+\theta h)$, $0<\theta<1$. 4

(b) If $u = \log(x^3+y^3+z^3-3xyz)$, show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{4}{x+y+z}$. 4

5. (a) If $u = f\left(\dfrac{y-x}{xy}, \dfrac{z-x}{zx}\right)$, prove that $x^2\dfrac{\partial u}{\partial x} + y^2\dfrac{\partial u}{\partial y} + z^2\dfrac{\partial u}{\partial z} = 0$. 4

(b) If $x\cos\alpha + y\sin\alpha = p$ touches the curve $\dfrac{x^m}{a^m} + \dfrac{y^m}{b^m} = 1$, show that $(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$. 4

6. (a) Find radius of curvature of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$ at any point θ. 4

(b) Find the asymptotes of the equation $(a+x)^2(b^2+x^2) = x^2y^2$. 4

7. (a) Expand e^x in ascending powers of $(x-1)$. 4

(b) Verify Rolle's theorem for $f(x) = x^3 - 6x^2 + 11x - 6$ in [1, 3]. 4

8. (a) Prove that $\frac{2x}{\pi} < \sin x < x$ for $x > 0$. 4

(b) Find the greatest and the least value of $2\sin x + \sin 2x$ in the interval $\left(0, \frac{3\pi}{2}\right)$. 2+2

9. (a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally. 4

(b) Find the points on the parabola $y^2 = 2x$ which is nearest to the point (3, 0). 4

10.(a) Find the values of a and b such that the function 5

$$f(x) = \begin{cases} x+\sqrt{2}a\sin x & , \quad 0 \le x \le \frac{\pi}{4} \\ 2x\cot x + b & , \quad \frac{\pi}{4} < x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & , \quad \frac{\pi}{2} < x \le \pi \end{cases}$$

is continuous for all values of x in the interval $0 \le x \le \pi$.

(b) If $u(x,\ y) = \cot^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{4}\sin 2u = 0$. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2020, held in 2021

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours　　　　Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Evaluate $\lim\limits_{x\to0}(1+3x)^{\frac{2}{x}}$

(b) Find $\lim\limits_{x\to2}\sqrt{x-2}$ if it exists.

(c) Show that $f(x)=2x^2+3x+5$ is continuous for any real number x.

(d) Find $\dfrac{dy}{dx}$ if $(\cos x)^y=(\sin y)^x$.

(e) If $y=\frac{x}{x+1}$, show that $y_5(0)=5!$.

(f) At what point is the tangent to the parabola $y=x^2$ parallel to the straight line $y=4x-5$.

(g) Find the points of extremum value of the function $f(x)=\sin x(1+\cos x)$ in $[0,2\pi]$.

(h) If $f(x,y)=x\log y$ then show that $f_{xy}=f_{yx}$.

(i) Find the asymptotes of the curve $x^3-6x^2y+11xy^2-6y^3+x+y+5=0$.

(j) Find the radius of curvature of the curve $xy=12$ at (3, 4).

2. (a) A function f is defined as follows: 4

$$f(x)=\begin{cases} x^2+ax &, \text{ if } 0\le x<1\\ 3-bx^2 &, \text{ if } 1\le x\le 2\end{cases}$$

If $\lim\limits_{x\to1}f(x)=4$, find the value of a and b.

(b) If $f, g : D \to \mathbb{R}$ are two functions such that $\lim\limits_{x \to c} f(x)$ and $\lim\limits_{x \to c} g(x)$ exists finitely, then prove that $\lim\limits_{x \to c} [f(x) + g(x)] = \lim\limits_{x \to c} f(x) . \lim\limits_{x \to c} g(x)$. 4

3. (a) State and prove Lagrange's mean value theorem. 5

(b) If $f(x, y) = \tan^{-1}\dfrac{y}{x} + \sin^{-1}\dfrac{y}{x}$, find the value of $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ at the point (1, 1). 3

4. (a) If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} + (m^2 - n^2)y_n = 0$. 5

(b) Find the radius of curvature of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ at any point θ. 3

5. (a) Show that the function f is continuous at $x = 1$ but not differentiable at $x = 1$ where 4

$$f(x) = \begin{cases} x+1 & , \text{ if } 0 \le x < 1 \\ 3-x & , \text{ if } 1 \le x \le 2 \end{cases}$$

(b) Find the points on the curve $y = 2x^3 - 15x^2 + 34x - 20$ where the tangents are parallel to the straight line $y + 2x = 0$. 4

6. (a) If $f(x, y) = \begin{cases} \dfrac{x^3 y}{x^2 + y^2} & , \ x^2 + y^2 \ne 0 \\ 0 & , \ x = 0, \ y = 0 \end{cases}$ 5

prove that $f_{xy}(0,0) \ne f_{yx}(0,0)$.

(b) Find the nature of double points of the curve $(2y + x + 1)^2 = 4(1-x)^5$. 3

7. (a) Determine the points of discontinuities of the function 4

$$f(x) = \begin{cases} \sin\frac{1}{x} & , \ x \le 0 \\ 2x & , \ 0 < x < 1 \\ 0 & , \ x = 1 \\ \frac{x^2-1}{x-1} & , \ 1 < x \end{cases}$$

(b) Prove that $\dfrac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. 4

8. (a) Determine the Taylor's series expansion of $f(x) = \cos x$. 5

(b) If a function f is differentiable on [0, 1] show that the equation $f(1) - f(0) = \dfrac{f'(x)}{2x}$ has at least one root in (0, 1). 3

9. (a) If $u = f(x,\ y)$ and $x = r\cos\theta$, $y = r\sin\theta$ then prove that 4

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) Show that the area of a rectangle inscribed in a circle is the maximum when it is a square. 4

10.(a) A function f is thrice differentiable on $[a,\ b]$ and $f(a) = 0 = f(b)$ and $f'(a) = 0 = f'(b)$. Prove that there is a number c in $[a,\ b]$ such that $f'''(c) = 0$. 3

(b) If $u = f(y-z,\ z-x,\ x-y)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 5

11.(a) Show that the radius of curvature at any point $(r,\ \theta)$ on the curve $r = a(1-\cos\theta)$ varies as $\sqrt{r}$. 5

(b) If $f(x) = 2|x| + |x-2|$, find $f'(1)$. 3

12.(a) Find the asymptotes of the following curve: 5

$$x = \frac{t^2}{1+t^3} \quad , \quad y = \frac{t^2+2}{1+t}$$

(b) If $\lim\limits_{x\to 0} \frac{\sin 2x + a\sin x}{x^3}$ be finite, find the value of 'a' and the limit. 3

13.(a) State and prove Leibnitz's theorem on successive differentiation. 5

(b) Find the radius of curvature of the curve $y = xe^{-x}$ at its maximum point. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2019

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours

Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Evaluate the right hand and left hand limits of the function $f(x)=\frac{|x|}{x}$ at the point $x=0$. Examine whether the function has a limit at 0.

(b) Find the points of discontinuity of the function $f(x)=\frac{x-1}{(x^2-1)(x-1)x}$.

(c) Verify Rolle's theorem for the function $f(x)=x^2-5x+10$ on $[2, 3]$.

(d) Investigate the extremum for the function $f(x)=2x^3-15x^2+42x+10$.

(e) Show that $\frac{\sin\alpha-\sin\beta}{\cos\alpha-\cos\beta}=\cot\theta$, where $0<\alpha<\theta<\beta<\frac{\pi}{2}$.

(f) Show that the function $f(x)=\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/4}+y^{1/4}}}$ is homogeneous and find its degree.

(g) Find the points on the curve $y=x^2+3x+4$, where the tangents pass through the origin.

(h) If $y=\sin(m\sin^{-1}x)$, show that $(1-x^2)y_2-xy_1+m^2y=0$.

2. (a) Show that the limit that $\lim_{x\to0}\sin\frac{1}{x}$ does not exist. 2

(b) If two functions f and g are continuous at a point c, then show that $f+g$ is also continuous at c. 3

(c) Discuss the continuity of the function $f(x)=|x-3|$ at $x=3$ and find $f'(3)$, if exists. 2+1

3. (a) If a function f is differentiable at some point c in its domain, then prove that it is also continuous at c. Give a suitable example to show that the converse of the above result is not true. 3

(b) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, then prove that $x\dfrac{\delta u}{\delta x} + y\dfrac{\delta u}{\delta y} = \sin 2u$. 3

(c) Find the equation of the normal to the curve $x^2 - y^2 = a^2$ at the point $(a\sqrt{2}, a)$. 2

4. (a) If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. 4

(b) State and prove Euler's theorem on homogeneous functions. 4

5. (a) Find the radius of curvature at $(\frac{1}{4}, \frac{1}{4})$ of the curve $\sqrt{x} + \sqrt{y} = 1$. 3

(b) State and prove Lagrange's mean value theorem. Write the geometrical interpretation of this theorem. 4+1

6. (a) Find the Taylor's series expansion of the function $f(x) = \sin x,\ x \in \mathbb{R}$. 5

(b) Determine the asymptotes of the curve $x = \dfrac{2t}{t^2-1},\ y = \dfrac{(1+t)^2}{t^2}$. 3

7. (a) Verify Rolle's theorem for the function $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$. 4

(b) If $f:\ \mathbb{R} \longrightarrow \mathbb{R}$ is a differentiable function such that $f'(x) = 0$ everywhere then show that $f(x)$ is a constant function on $\mathbb{R}$. 4

8. (a) If $f(h) = f(0) + hf'(0) + \dfrac{h^2}{2!}f''(\theta h),\ 0 < \theta < 1$, find θ when $h = 1$ and $f(x) = (1-x)^{3/2}$. 3

(b) Discuss maxima and minima of the function $f(x) = (\frac{1}{x})^x,\ x > 0$, if there be any. 5

9. (a) If $u = f(x, y),\ x = r\cos\theta,\ y = r\sin\theta$, then prove that 6

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) If $f(x, y) = \dfrac{xy(x^2-y^2)}{x^2+y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ then find $f_x(0, 0)$ and $f_y(0, 0)$. 1+1

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2018

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours | Full Marks: 50

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Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) A function $f(x)$ is defined as follows: 2

$$f(x) = |x-2|+1$$

Examine whether $f'(2)$ exists.

(b) Examine whether $f(x,y) = x^{-1/3}y^{4/3}\cos\left(\frac{y}{x}\right)$ is a homogeneous function of x and y. If so, find its degree. 2

(c) Find the value of $\frac{d^n}{dx^n}\{\sin(ax+b)\}$ 2

(d) Is Rolle's theorem applicable to the function $|x|$ in the interval $[-1, 1]$? Justify your answer. 2

(e) Find the radius of curvature at the origin for the curve $x^3+y^3-2x^2+6y=0$. 2

(f) Find the asymptotes parallel to co-ordinate axes of the curve $(x^2+y^2)x-ay^2=0$. 2

(g) If $e^{a\sin^{-1}x} = a_0+a_1x+a_2x^2+\ldots\ldots$, then find the value of a_2. 2

(h) Evaluate: $\lim\limits_{x\to 0}(\cos x)^{\cot x}$ 2

2. (a) If $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exists finitely for two functions f and g, then prove that $\lim\limits_{x\to a}\{f(x)+g(x)\} = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$ 3

(b) Using ε-s definition (Cauchy's definition) show that the function f defined by, 3

$$f(x) = x^2, \quad x \text{ is rational}$$
$$= -x^2, \quad x \text{ is irrational}$$

is continuous at 0.

(c) Find the co-ordinates of the points on the curve $y = x^2 - 8x + 5$ at which the tangents pass through the origin. 2

3. (a) If $f(x) = \begin{cases} x+1 \;, & \text{when } x \le 1 \\ 3 - ax^2 \;, & \text{when } x > 1 \end{cases}$ 3

then find the value of a for which f is continuous at $x = 1$.

(b) Find the Taylor series expansion of $f(x) = \sin x$. 5

4. (a) If $u(x,y) = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, $x \ne y$, apply Euler's theorem to find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ and hence show that $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$ 5

(Assume $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$)

(b) If $y = \dfrac{x}{x+1}$, find y_n (where y_n is the n-th differential coefficient of y w.r.t x) and hence find $y_7(0)$. 3

5. (a) If $f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} \;, & (x,y) \ne (0,0) \\ 0 \;, & (x,y) = (0,0) \end{cases}$ 4

Show that $f_{xy}(0,0) \ne f_{yx}(0,0)$.

(b) Find the asymptotes of the cubic $y^3 + x^2y + 2xy^2 - y + 1 = 0$ 4

6. (a) State and prove Cauchy's Mean Value Theorem. 5

(b) If $\lim\limits_{x \to 0} \dfrac{\sin 2x + a\sin x}{x^3}$ is finite, find a and the value of the limit. 3

7. (a) Find the radius of curvature at any point (r, θ) for the curve $r = a(1 - \cos\theta)$. Hence show if ρ_1 and ρ_2 be the radii of curvature at the extremities of any chord of this cardioid which pass through the pole; then prove that $\rho_1^2 + \rho_2^2 = \dfrac{16a^2}{9}$ 5

(b) Show that the normal to the curve $3y = 6x - 5x^3$ drawn at the point $\left(1, \frac{1}{3}\right)$ passes through the origin. 3

8. (a) If $H = f(y-z,\ z-x,\ x-y)$, then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$ 3

(b) Verify Rolle's theorem for the function $f(x) = x^2 + \cos x$ on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$. 2

(c) If $V = x\sin^{-1}\left(\frac{y}{x}\right) + y\tan^{-1}\left(\frac{x}{y}\right)$, find the value of $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}$ at (1, 1) 3

9. (a) Show that at any point of the curve $by^2 = (x+a)^3$, the subnormal varies as the square of the subtangent. 4

(b) Prove that of all the rectangular parallelopiped of the same volume, the cube has the least surface area. 4

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